**STATISTICS ASSIGNMENT\_10**

**1. A chicken lays n eggs. Each egg independently does or doesn’t hatch, with probability p of hatching. For each egg that hatches, the chick does or doesn’t survive (independently of the other eggs), with probability s of survival. Let N ⇠ Bin(n, p) be the number of eggs which hatch, X be the number of chicks which survive, and Y be the number of chicks which hatch but don’t survive (so X + Y = N). Find the marginal PMF of X, and the joint PMF of X and Y . Are they independent?**

(a) The marginal PMF of X can be found as follows:

P(X = x) = P(X = x, Y = N - x) = sum over all y of P(X = x, Y = y)

= sum over all y of P(X = x | Y = y) \* P(Y = y)

= sum over all y of P(X = x | N = x + y) \* P(N = x + y)

= sum over all y of s^x \* (1 - s)^(y) \* p^(x + y) \* (1 - p)^(n - x - y) \* n!/(x + y)!(n - x - y)!

(b) The joint PMF of X and Y can be found as:

P(X = x, Y = y) = P(X = x | N = x + y) \* P(N = x + y)

= s^x \* (1 - s)^y \* p^(x + y) \* (1 - p)^(n - x - y) \* n!/(x + y)!(n - x - y)!

(c) X and Y are not independent because the occurrence of one event affects the probability of the other event. For example, if X = x, then Y = N - x, which affects the probability of Y.